Torsion of thin-walled closed sections
(Strength of Materials - II, Final Exam-43-5)

Problem:

Determine the shear stresses, angle of twist and torsional rigidity, $GJ$, for the thin-wall tubular member whose cross section is shown in figure. The shear modulus is $G$.

Solution:

The shear stress through the wall-thickness is
\[ \tau = \frac{T}{2tA} \]

the shaded area is
\[ A = \frac{\pi}{2} (6t_0)^2 + (11t_0)(13t_0) = 209.37t_0^2 \]

the average shear stresses are
\[ \tau_1 = \frac{T}{2t_0} \frac{1}{209.37t_0^2} = \frac{1}{418.74} \frac{T}{t_0} \]
\[ \tau_2 = \frac{T}{2} \frac{1}{209.37t_0^2} = \frac{1}{837.48} \frac{T}{t_0} \]

The angle of twist is
\[ \phi = TL \frac{\int ds}{4A^2G_C \int t} \]
\[ \int ds = \sum_i \left( \frac{\Delta s_i}{t} \right) = \frac{\pi(6.5t_0)}{t_0} + \frac{2(11t_0)}{t_0} + \frac{13t_0}{2t_0} = 48.92 \]
\[ \phi = \frac{TL}{4A^2G_C} \frac{\int ds}{t} = 48.92 \frac{TL}{4 \times (209.37t_0^2)^2 \times G} \]
\[ \phi = \frac{1}{3584.2} \frac{TL}{G \ t_0^4} \]

It is known that
\[ \phi = \frac{TL}{GJ} \]

therefore
\[ \frac{1}{3584.2} \frac{TL}{G \ t_0^4} = \frac{TL}{GJ} \]
\[ GJ = 3584.2 \ G \ t_0^4 \]

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