Residual stresses in beams
(Strength of Materials - II, Midterm Exam-42-5)

Problem:

1. Elasto-plastic beam in pure bending

A rectangular box beam with height $h = 12$ in. and width $b = 8$ in. has a constant wall thickness $t = 0.50$ in. It is made of structural steel with $\sigma_Y = 36$ ksi and $E = 30 \times 10^3$ ksi. Calculate numerical values for the yield moment $M_Y$, the plastic moment $M_P$, and the residual stress distribution after unloading the plastic moment.

Solution:

The centroidal principal moment of inertia of the beam section is

$$I_{zz} = \frac{8 \times 12^3}{12} - \frac{7 \times 11^3}{12} = 375.58 \text{ in}^4$$

Onset of yielding

The maximum elastic moment is:

$$\sigma_Y = \frac{M_Y \times (\frac{h}{2})}{I_{zz}}$$

$$36 \times 10^3 = \frac{M_Y \times (6)}{375.58}$$

$$M_Y = 2254 \text{ kip-in}$$

Since $F_1 = F_4$ and $F_2 = F_3$ the plastic moment is

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2. Fully plastic case

\[ M_p = \sigma_Y \sum (A_i \times \bar{y}_i) = F_1 \times d_1 + F_2 \times d_2 \]

\[ M_p = 2 \times 36 \times 10^3 \times (0.5 \times 8) \times (6 - 0.25) + 2 \times 2 \times 36 \times 10^3 \times \left(0.5 \times \frac{11}{2}\right) \times \frac{11}{4} \]

\[ M_p = 2.745 \text{ kip-in} \]

The maximum reverse stress is

\[ \sigma' = \frac{M_p \times \left(\frac{h}{2}\right)}{I_{zz}} = \frac{2.745 \times 10^3 \times 6}{375.58} \]

\[ \sigma' = 43.852 \text{ ksi} \]

Residual stress distribution

The residual stresses at some locations are

\[ \sigma_{R_A} = \sigma_Y + \sigma'_A = -36 + 43.852 = 7.852 \text{ ksi} \]

\[ \sigma_{R_B} = \sigma_Y + \sigma'_B = -36 + 43.852 \times \frac{11}{12} = 4.1977 \text{ ksi} \]

\[ \sigma_{R_C} = \sigma_Y + \sigma'_C = -36 + 0 = -36 \text{ ksi} \]

The zero residual stress occurs at

\[ \frac{7.852 + 36}{36} = \frac{6}{y}, \quad y = 4.9256 \text{ in} \]