Problem:

1. T beam in pure bending

The cross section of a beam is a T with the dimensions shown in Figure. The moment at the section is \( M = 4 \) kip-ft. Determine (a) the location of the neutral axis of the cross section, (b) the moment of inertia with respect to the neutral axis, and (c) the maximum tensile stress and the maximum compressive stress on the cross section. (d) the resultant compressive force \( F_C \), the resultant tensile force \( F_T \), and the distance that separates them.

Solution:

The centroid of the cross section is

\[
\bar{y} = \frac{\sum (A \times \bar{y})}{\sum A} = \frac{(1 \times 5) \times 5.5 + (5 \times 1) \times 2.5}{1 \times 5 + 5 \times 1} = 4 \text{ in}
\]

the centroidal moment of inertia is

\[
I_{zz} = \frac{5 \times 1^3}{12} + (5 \times 1) (5.5 - 4)^2 + \frac{1 \times 5^3}{12} + (1 \times 5) (2.5 - 4)^2 = 33.333 \text{ in}^4
\]

The normal stresses are

\[
\sigma_A = \frac{M_x \times y_A}{I_{zz}} = \frac{4 \times 10^3 \times (12 \times 2)}{33.333} = 2880.0 \text{ psi} \quad (-)
\]
\[
\sigma_B = \frac{M_x \times y_B}{I_{zz}} = \frac{4 \times 10^3 \times (12 \times 4)}{33.333} = 5760.1 \text{ psi} \quad (+)
\]
\[
\sigma_C = \frac{1}{2} \sigma_A = \frac{2880.0}{2} = 1440 \text{ psi} \quad (-)
\]

The resultant compressive force is

\[
F_c = \int_A \sigma_{xx} \, dA
\]

Dr. M. Kemal Apalak
\[ F_C = 1440 \times (5 \times 1) + \frac{1}{2} \times 1440 \times (5 \times 1) + \frac{1}{2} \times 1440 \times (1 \times 1) \]

\[ F_C = 11520.0 \text{ lb (–)} \]

the resultant tensile force is

\[ F_T = \frac{1}{2} \times 5760 \times (1 \times 4) \]

\[ F_T = 11520.0 \text{ lb (+)} \]

2. The stress distribution and resultant forces

The distance between the resultant compressive and tensile forces \( d \) is

\[ d = y_C + y_T = y_C + \frac{2}{3} \times 4 \]

\[ \sum M = F \times d \]

\[ 4000 \times 12 = 11520 \times d \]

\[ d = 4.167 \text{ in} \]

\[ y_C = 4.167 - \frac{2}{3} \times 4 \]

\[ y_C = 1.50 \text{ in} \]