A thin walled tube has been fabricated by bending a metal plate of thickness \( t \) into a cylinder of radius \( c \) and bonding together the edges of the plate. A torque \( T \) is then applied to the tube, producing a shearing stress \( \tau_1 \) and an angle of twist \( \phi_1 \).

Denoting by \( \tau_2 \) and \( \phi_2 \), respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios \( \tau_2 / \tau_1 \) and \( \phi_2 / \phi_1 \) in terms of the ratio \( c/t \).

In case the bond fails the section forms a thin walled open section the maximum shear stress is

\[
\tau_2 = \frac{T}{c_1 ab^3}
\]

where \( a = 2\pi c \) and \( b = t \)

\[
a = \frac{2\pi c}{b} = \frac{2\pi c}{t}
\]

therefore \( c_1 = \frac{1}{3} \)

\[
\tau_2 = \frac{T}{\frac{1}{3}(2\pi c)t^2}
\]

\[
\tau_2 = \frac{3}{2\pi ct} T
\]

The polar moment of inertia of the thin walled tube is

\[
J = \int r^2 dA = \int c^2 dA \quad \text{where} \quad dA = ct d\theta
\]

\[
J = \int c^2 (ct d\theta) = c^3 t \int_0^{2\pi} d\theta
\]

\[
J = 2\pi c^3 t
\]

and the maximum shear stress is

\[
\tau_1 = \frac{Tc}{J} = \frac{Tc}{2\pi ct^3}
\]

\[
\tau_1 = \frac{T}{2\pi ct^3}
\]

The ratio of the shear stresses is

\[
\frac{\tau_2}{\tau_1} = \left( \frac{3}{2\pi ct} \right) \left( \frac{2\pi ct}{T} \right)
\]

\[
\frac{\tau_2}{\tau_1} = 3 \left( \frac{c}{t} \right)
\]

In case of the thin walled open section the angle of twist is

\[
\phi_2 = \frac{TL}{c_2 ab^3 G}
\]

where \( c_2 = \frac{1}{3} \)

\[
\phi_2 = \frac{TL}{\frac{1}{3}(2\pi c)t^3 G}
\]

\[
\phi_2 = \frac{3}{2\pi ct^3 G}
\]

In case of the thin walled tube the angle of twist is

\[
\phi_1 = \frac{TL}{GJ}
\]

The ratio of the angles of twist is

\[
\frac{\phi_2}{\phi_1} = \left( \frac{3}{2\pi ct^3 G} \right) \left( \frac{G}{TL} \right) \left( \frac{2\pi ct}{T} \right)
\]

\[
\frac{\phi_2}{\phi_1} = 3 \left( \frac{c}{t} \right)^2
\]

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