The design specifications for the gear-and-shaft system shown require that the same diameter be used for both shafts and that the angle through which pulley A will rotate when subjected to a 200 Nm torque $T_A$ while pulley D is held fixed will not exceed 7.5°. Determine the required diameter of the shafts if both shafts are made of a steel with $G = 77$ GPa and $\tau_{all} = 80$ MPa.

\[ \phi_B = \phi_C \phi_C = \frac{\Gamma_B}{\Gamma_C} \phi_B \]

\[ \phi_C = \frac{50}{125} \phi_B = \frac{10}{25} \phi_B \quad (1) \]

\[ C_{\text{max}} = \frac{T_A C}{J} = \frac{T_C C}{T_C C} = \frac{2T_A}{\pi C^3} \]

\[ 80 = \frac{2(200 \times 10^3)}{\pi C^3} \Rightarrow C = 11.68 \text{ mm} \]

\[ \frac{T_A}{\Gamma_B} = \frac{\Gamma_C}{\Gamma_C} \Rightarrow T_C = \frac{\Gamma_C}{\Gamma_B} T_A = \frac{125(200)}{50} \]

\[ T_C = 500 \text{ N} \cdot \text{m} \]

\[ C_{\text{max}} = \frac{T_C C}{J} = \frac{T_C C}{\pi C^4} = \frac{2T_C}{\pi C^3} \]

\[ C_{\text{max}} = \frac{2 \times 500 \times 10^3}{\pi (11.675)^3} = 200 \text{ MPa} > 80 \text{ MPa} \]

Therefore,

\[ 80 = \frac{2T_C}{\pi C^3} \Rightarrow 80 = \frac{2}{\pi} \frac{500 \times 10^3}{C^3} \]

\[ C = 15.847 \text{ mm} \quad \text{and} \quad d = 2C = 31.69 \text{ mm} \]

\[ \phi_C/0 = \frac{T_C C}{G J} = \frac{500 \times 0.16}{77 \times 10^9 \times \frac{\pi}{2} (0.015847)^4} \]

\[ \phi_C/0 = 0.09933 \text{ rad} \left( \frac{180}{\pi} \right) = 2.253^\circ < 7.5^\circ \]

\[ \phi_C/0 = \phi_C/0 = \frac{10}{25} \phi_B = 2.253^\circ \Rightarrow \phi_B = 56.3^\circ < 7.5^\circ \]

For both shafts:

\[ C = 15.847 \text{ mm} \quad d = 31.69 \text{ mm} \]

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