Plane stress
(Strength of Materials - I, Final Exam-41-3)

Problem:

1. Superposition of two different plane stress states

Determine a) the principal planes and the principal stresses and b) the maximum shearing stresses, and c) draw the Mohr’s circle for the state of plane stress resulting from the superposition of the two states of plane stress shown.

Solution:

The Case I can be transformed to the stress state in the coordinate axes of the Case II for $\theta = 30^\circ$ as follows:

\[
\begin{align*}
\sigma_{xx} &= \sigma'_{xx} + \sigma'_{yy} + \frac{\sigma'_{xx} - \sigma'_{yy}}{2} \cos 2\theta + \sigma'_{xy} \sin 2\theta \\
\sigma_{xx} &= 0 + 0 + 50 \times \sin 60 \\
\sigma_{xx} &= 43.3 \text{ MPa} \\
\sigma_{yy} &= \frac{\sigma'_{xx} + \sigma'_{yy} - \frac{\sigma'_{xx} - \sigma'_{yy}}{2} \cos 2\theta - \sigma'_{xy} \sin 2\theta}{2} \\
\sigma_{yy} &= 0 - 0 - 50 \times \sin 60 \\
\sigma_{yy} &= -43.3 \text{ MPa} \\
\sigma_{xy} &= \frac{\sigma'_{xy} - \frac{\sigma'_{xx} - \sigma'_{yy}}{2} \sin 2\theta + \sigma'_{xy} \cos 2\theta}{2} \\
\sigma_{xy} &= 0 + 50 \times \cos 60 \\
\sigma_{xy} &= 25 \text{ MPa}
\end{align*}
\]

The superposition of the transformed Case - I and Case - II yields

\[
\begin{align*}
\sigma_{xx} &= 43.3 + 75 = 118.3 \text{ MPa} \\
\sigma_{yy} &= -43.3 + 100 = 56.7 \text{ MPa} \\
\sigma_{xy} &= 25 + 50 = 75 \text{ MPa}
\end{align*}
\]

The average stress is

\[
\sigma_{ave} = \frac{\sigma_{xx} + \sigma_{yy}}{2} = \frac{118.3 + 56.7}{2} = 87.5 \text{ MPa}
\]

The difference of the normal stresses is

\[
\sigma_{dif} = \frac{\sigma_{xx} - \sigma_{yy}}{2} = \frac{118.3 - 56.7}{2} = 30.8 \text{ MPa}
\]

The radius of the Mohr’s circle is

\[
R = \sqrt{\sigma_{dif}^2 + \sigma_{xy}^2} = \sqrt{30.8^2 + 75^2} = 81.08 \text{ MPa}
\]

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2. Equivalent stress system

The principal stresses are

\[ \sigma_{a,b} = \sigma_{\text{ave}} \pm R = 87.5 \pm 81.08 \]
\[ \sigma_a = 168.58 \text{ MPa} \]
\[ \sigma_b = 6.42 \text{ MPa} \]

The principal directions are

\[ \tan 2\theta_a = \frac{\sigma_{xy}}{\sigma_{\text{dif}}} = \frac{75}{30.8}, \quad \theta_a = 33.84^\circ \]
\[ \theta_b = 33.84^\circ + 90 = 123.84^\circ \]

The maximum in-plane shearing stress is

\[ (\tau_{\text{max}})_{\text{in-plane}} = R = 81.08 \text{ MPa} \]

and the normal stress in these planes is

\[ \sigma_n = \sigma_{\text{ave}} = 87.5 \text{ MPa} \]

The corresponding directions are

\[ \theta_d = \theta_a + 45 = 33.84 + 45, \quad \theta_d = 78.84^\circ \]
\[ \theta_e = 78.84^\circ + 90 = 168.84^\circ \]
3. Mohr's representation of the plane stress state

![Mohr's circle diagram]

4. Representation of the principal planes in the physical plane

![Principle planes diagram]