The strains determined by the use of a rosette attached as shown to the surface of a structural component are $\varepsilon_1 = 110 \mu$, $\varepsilon_2 = 212.5 \mu$, $\varepsilon_3 = 240 \mu$. Determine (a) the orientation and magnitude of the principal strains in the plane of the rosette, (b) the maximum in-plane shearing strain, (c) the maximum shearing strain. (Use $\nu = 0.3$)

The strains are

$$\varepsilon_1 = \varepsilon_{xx} \cos^2 \theta_1 + \varepsilon_{yy} \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1$$

For $\theta_1 = 45^\circ$

$$110 = 212.5 \cos^2 45 + \varepsilon_{yy} \sin^2 45 + \gamma_{xy} \sin 45 \cos 45$$

$$7.5 = \varepsilon_{yy} + \gamma_{xy} \quad (1)$$

$$\varepsilon_3 = \varepsilon_{xx} \cos^2 \theta_3 + \varepsilon_{yy} \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3$$

For $\theta_3 = 135^\circ$

$$240 = 212.5 \cos^2 135 + \varepsilon_{yy} \sin^2 135 + \gamma_{xy} \sin 135 \cos 135$$

$$133.75 = 0.5 \varepsilon_{yy} - 0.5 \gamma_{xy}$$

$$267.5 = \varepsilon_{yy} - \gamma_{xy} \quad (2)$$

Solving Equations (1 and 2) for $\varepsilon_{yy}$ and $\gamma_{xy}$

$$\varepsilon_{yy} = 137.5 \mu$$

and

$$\gamma_{xy} = -130 \mu$$

The average of the normal strains is

$$\varepsilon_{ave} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} = \frac{212.5 + 137.5}{2} = 175 \mu$$

The difference of the normal strains is

$$\varepsilon_{dif} = \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} = \frac{212.5 - 137.5}{2} = 37.5 \mu$$

The radius of Mohr’s circle is

$$R = \sqrt{\varepsilon_{dif}^2 + \varepsilon_{yy}^2} = \sqrt{37.5^2 + \left(-\frac{130}{2}\right)^2} = 75.04 \mu$$

The principal strains are

$$\varepsilon_a = \varepsilon_{ave} + R = 175 + 75.04 = 250.04 \mu$$

$$\varepsilon_b = \varepsilon_{ave} - R = 175 - 75.04 = 99.96 \mu$$

The principal directions are

$$\tan 2\theta_a = \frac{\varepsilon_{xy}}{\varepsilon_{dif}} = -\frac{65}{37.5}, \quad \theta_a = 30^\circ \quad (CW)$$

The maximum in-plane shearing strain is

$$\gamma_{max} = \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} = \frac{250.04 - 99.96}{2}$$

$$\gamma_{max} = 75 \mu, \quad \gamma_{max} = 150 \mu$$

Since $\sigma_z = 0$ on the surface, the third principal strain is

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{0.3}{1-0.3}(250.04 + 99.96)$$

$$\varepsilon_c = -150 \mu$$

The maximum shearing strain is

$$\gamma_{max} = \frac{\varepsilon_a - \varepsilon_c}{2} = \frac{250.04 - (-150)}{2}$$

$$\gamma_{max} = 200.02 \mu$$