For a state of plane stress it is known that for each of two given orientations of the coordinate axes, the normal and shearing stresses are directed as shown and the magnitudes of the normal stresses \( \sigma_{xx}, \sigma_{yy}, \) and \( \sigma_{xy} \) are, respectively, 75 MPa, 15 MPa, and 90 MPa. Determine (a) the principal planes and principal stresses, (b) the maximum in-plane shearing stress and (c) the maximum shearing stress.

\[
\begin{align*}
\sigma_{xx}' & = 75 \text{ MPa}, \\
\sigma_{yy}' & = 15 \text{ MPa}, \\
\sigma_{xy}' & = 90 \text{ MPa} \\
\sigma_{xx}' & = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\theta + \sigma_{xy} \sin 2\theta \\
\text{ave} & = \frac{\sigma_{xx} + \sigma_{yy}}{2} = \frac{75 + 15}{2} = 45 \text{ MPa} \\
\text{diff} & = \frac{\sigma_{xx} - \sigma_{yy}}{2} = \frac{75 - 15}{2} = 30 \text{ MPa} \\
\theta & = 45 + 30 \cos 60 + 34.64 \sin 60 \Rightarrow \sigma_{xy} = 34.64 \text{ MPa} \\
\frac{\sigma_{xy}}{\sigma_{xx}} & = \frac{34.64}{45} \Rightarrow \theta_a = 24.55^\circ \\
\theta_b & = 114.55^\circ
\end{align*}
\]

The principal directions \( \tan 2\theta_a = \frac{\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}} = \frac{34.64}{30} \Rightarrow \theta_a = 24.55^\circ \)

The maximum in-plane shearing stress (see Figure I)

\( (\tau_{\text{max}})_{\text{in-plane}} = 46 \text{ MPa} \)

The maximum shearing stress \( \tau_{\text{max}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \)

\( \tau_{\text{max}} = \frac{91 - (-1)}{2} = 46 \text{ MPa} \)

(see Figure II)