A compressed-air tank is supported by two cradles as shown: one of the cradles is designed so that it does not exert any longitudinal force on the tank. The cylindrical body of the tank is fabricated from a 12-mm steel plate by butt-welding along a helix, which forms an angle of 30° with a transverse plane. The end caps are spherical and have a uniform wall thickness of 10 mm. For an internal gage pressure of 1.5 MPa, determine (a) the normal stress and the maximum shearing stress in the spherical caps, (b) the stresses in directions perpendicular and parallel to the helical weld.

(a) Spherical cap

\[ \sigma_1 = \sigma_2 = \frac{pr}{2t} \quad p = 1.5 \text{ MPa} \]
\[ t = 10 \text{ mm} \]

\[ \sigma_1 = \sigma_2 = \frac{1.5 \left( \text{N/mm}^2 \right) \times \left( \frac{600}{2} \right) \text{ mm}}{2 \times 10 \text{ (mm)}} \]
\[ \sigma_1 = \sigma_2 = 27.5 \text{ MPa} \] (normal stresses)

\[ \tau_{\text{max}} = \frac{\sigma_1 - \sigma_3}{2} = \frac{27.5 - 0}{2} \Rightarrow \tau_{\text{max}} = 11.25 \text{ MPa} \]
(Max. shear stress)

(b) Cylindrical body of the tank.

\[ \sigma_1 = \frac{pr}{t} = \frac{1.5 \left( \text{N/m}^2 \right) \times \left( \frac{600}{2 \text{ mm}} \right)}{12 \text{ (mm)}} \]
\[ \sigma_1 = 37.5 \text{ MPa} \]

\[ \sigma_2 = \sigma_3 = \frac{37.5}{2} \Rightarrow \sigma_2 = 18.75 \text{ MPa} \]

Stresses at the weld.

\[ \sigma_{\text{ave}} = \frac{\sigma_1 + \sigma_2}{2} = \frac{37.5 + 18.75}{2} \Rightarrow \sigma_{\text{ave}} = 28.125 \text{ MPa} \]

\[ R_1 = \frac{\sigma_1 - \sigma_2}{2} = \frac{37.5 - 18.75}{2} \Rightarrow R_1 = 9.375 \text{ MPa} \]

\[ 2\theta = 2 \times 30 \Rightarrow 2\theta = 60^\circ \]

\[ \sigma_n = \sigma_{\text{ave}} - R_1 \cos 2\theta = 28.125 - 9.375 \cos 60 \]
\[ \sigma_n = 23.44 \text{ MPa} \]
\[ \tau_0 = -R_1 \tau_{22} = -9.375 \sin 60 \]

\[ \tau_S = -8.119 \text{ MPa} \]

\[ \tau_{\text{max}} = \frac{\sigma_1 - \sigma_3}{2} = \frac{37.5 - 0}{2} \Rightarrow \tau_{\text{max}} = 18.75 \text{ MPa}. \]

Normal stress on this plane:

\[ \sigma_n = \frac{\sigma_1}{2} = \frac{37.5}{2} = 18.75 \text{ MPa}. \]

\[ \sigma_{\text{ave}} = 28.125 \]

\[ \sigma_{\text{avg}} = \sigma_{\text{ave}} + R_1 \cos 2\theta = 28.125 + 9.375 \cos 60 \]

\[ \sigma_{\text{avg}} = 32.81 \text{ MPa} \]