A 450,000 N force is applied to member AG which is pinned at A (Figure 2) and which may be considered a rigid body. (a) Find the forces in members DB and FC assuming linear, elastic behavior of these members. Member DB has a modulus of elasticity of 7\times10^{10} \text{ Pa}, and member FC has modulus of elasticity of 1.75\times10^{11} \text{ Pa}. The cross sectional area of each vertical member is 6.25\times10^{-5} \text{ m}^2. (b) If the yield stress for member BD is 4.2\times10^8 \text{ Pa} and that of member CF is 7\times10^8 \text{ Pa} what is the maximum vertical movement of point G for linear elastic behavior of the vertical members? (c) What force is required to cause a vertical movement of point G of 1.5 times the value above? (Neglect the weights of the members, and assume elastic, perfectly plastic behavior.)

\begin{eqnarray*}
\text{(a) The equilibrium condition} \\
\Sigma M_A = 0 \quad 3F_{BD} + 6F_{CF} - 450000 \times 9 = 0 \Rightarrow F_{BD} + 2F_{CF} = 135000 \tag{1} \\
\text{The compatibility relation} \\
\frac{\Delta BD}{\Delta CF} = \frac{3}{6} \Rightarrow \Delta BD = 0.5 \Delta CF \tag{2} \\
\text{The stress-deformation relation} \\
\left(\frac{F L}{A E}\right)_{BD} = 0.5 \left(\frac{F L}{A E}\right)_{CF} \tag{3} \\
\text{since } l_{BD} = l_{CF} \text{ and } A_{BD} = A_{CF} \\
F_{BD} = 0.5 \frac{E_{BD}}{E_{CF}} F_{CF} = 0.5 \frac{7 \times 10^{10}}{1.75 \times 10^{11}} F_{CF} \Rightarrow F_{BD} = 0.2 F_{CF} \tag{4} \\
\text{Eqs (1) and (4) yield} \\
0.2 F_{CF} + 2 F_{CF} = 135000 \Rightarrow F_{CF} = 613636 \text{ N} \\
F_{BD} = 0.2 (613636) \Rightarrow F_{BD} = 122727 \text{ N} \tag{5} \\
\text{(b) The maximum forces for elastic case} \\
(F_{CF} y)_{y} = (6y)_{CF} \times A_{CF} = 7 \times 10^{8} \times 6.25 \times 10^{-5} = 43750 \text{ N} \\
(F_{BD} y)_{y} = (6y)_{BD} \times A_{BD} = 4.2 \times 10^{8} \times 6.25 \times 10^{-5} = 26250 \text{ N} \\
\text{Since } F_{BD} = 0.2 F_{CF} \text{ the member CF is critical.} \\
\Delta CF = \left(\frac{F L}{A E}\right)_{CF} = \frac{43750 \times 6}{6.25 \times 10^{-5} \times 1.75 \times 10^{11}} = 0.024 \text{ m} \\
\end{eqnarray*}
The compatibility condition
\[
\frac{\Delta cf}{\Delta c} = \frac{6}{9} \Rightarrow \Delta c = 1.5 \Delta cf = 1.5 \times 0.024 \Rightarrow \Delta c = 0.036 \text{ m}.
\]

c) \( \Delta c = 1.5 \times 0.036 = 0.054 \text{ m}. \)

\[
\Delta cf = \frac{6}{9} \Delta c = \frac{2}{3} (0.054) \Rightarrow \Delta cf = 0.036 \text{ m}
\]

The equilibrium condition
\[
3 F_{BD} + 6 F_{CF} = 9 F \Rightarrow F_{BD} + 2 F_{CF} = 3 F
\]

\[F_{CF} = 43750 \text{ N} \text{ (Member CF yields)}\]

\[F_{BD} = 0.2 F_{CF} = 0.2(43750) = 8750 \text{ N} \text{ (Member BD is elastic)}\]

\[F_{BD} + 2(43750) = 3F \Rightarrow F_{BD} + 87500 = 3F\]

The compatibility condition
\[
\Delta BD = \frac{1}{3} \Delta c = \frac{0.054}{3} = 0.018 \text{ m}
\]

\[
\Delta BD = \left( \frac{FL}{AE} \right)_{BD} \Rightarrow F_{BD} = \left( \frac{AE}{L} \right)_{BD} \Delta BD = 0.018 \left( \frac{6.25 \times 10^{-5} \times 7 \times 10^{10}}{6} \right)
\]

\[F_{BD} = 13125 \text{ N and the equilibrium condition yields}\]

\[87500 + 13125 = 3F \Rightarrow F = 33541.7 \text{ N}\]