A metallic bar is made of two linearly elastic materials, material 1 and material 2, that are bonded together at their interface. Assume that $E_2 > E_1$. Determine the distribution of normal stress that must be applied at each end if the bar is to undergo axial deformation, and determine the location of the point in the cross section where the resultant force $P$ must act. Express your answers in terms of $P$, $E_1$, $E_2$, and the dimensions of the bar.

Normal strain: $\varepsilon(x) = \varepsilon = \text{constant}$

Normal stresses:

$\sigma_{x1} = E_1 \varepsilon$ and $\sigma_{x2} = E_2 \varepsilon$

Resultant forces:

$P = E_1 E A_1 = E_1 E \left(\frac{bh}{3}\right)$
$P = E_2 E A_2 = E_2 E \left(\frac{2bh}{3}\right)$

Equilibrium condition: $\Sigma F_x = 0$  

$P = P_1 + P_2$

$P = E_1 E \left(\frac{bh}{3}\right) + E_2 E \left(\frac{2bh}{3}\right) = \frac{bh}{3} E (E_1 + 2E_2)$

Normal strain becomes:

$\varepsilon = \frac{3P}{bh} \frac{1}{E_1 + 2E_2}$

Normal forces:

$P_1 = E_1 \left(\frac{bh}{3}\right) \frac{3P}{bh} \frac{1}{E_1 + 2E_2} = \frac{E_1 P}{E_1 + 2E_2}$

$P_2 = E_2 \left(\frac{2bh}{3}\right) \frac{3P}{bh} \frac{1}{E_1 + 2E_2} = \frac{2E_2 P}{E_1 + 2E_2}$

Normal stresses:

$\sigma_{x1} = \frac{P_1}{A_1} = \frac{E_1 P}{E_1 + 2E_2} \frac{1}{\left(\frac{bh}{3}\right)} = \frac{3P E_1}{bh(E_1 + 2E_2)}$

$\sigma_{x2} = \frac{P_2}{A_2} = \frac{2E_2 P}{E_1 + 2E_2} \frac{1}{\left(\frac{2bh}{3}\right)} = \frac{3P E_2}{bh(E_1 + 2E_2)}$
\[ z \begin{bmatrix} M_A = 0 \end{bmatrix} \quad \frac{5h}{6} p_1 + \frac{h}{3} p_2 = P \quad y_p \]

\[ y_p = \frac{1}{P} \left[ \frac{5h}{6} p_1 + \frac{h}{3} p_2 \right] = \frac{h}{3P} \left[ \frac{5}{2} p_1 + p_2 \right] \]

\[ y_p = \frac{h}{3P} \left[ \frac{5}{2} \frac{E_1 P}{E_1 + 2E_2} + \frac{2E_2 P}{E_1 + 2E_2} \right] \]

\[ y_p = \frac{h}{3P} \left( \frac{5E_1 + 4E_2}{E_1 + 2E_2} \right) \]

\[ y_p = \frac{h}{2} \quad \text{for } E_1 = E_2 \]